Conversion between equatorial and helioprojective coordinates

Ivica Skokić*[1,2,3]

[1] *Astronomical Society Anonymus, Braće Radića 34, 31550 Valpovo, Croatia*
[2] *Hvar Observatory, Faculty of Geodesy, University of Zagreb, Kačićeva 26, HR-10000 Zagreb, Croatia*
[3] *Astronomical Institute, Czech Academy of Sciences, Fričova 298, 251 65 Ondřejov, Czech Republic*

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Abstract

Approximate and exact methods of conversion between equatorial (sky) and helioprojective (solar x, y) coordinates are presented and discussed.

1 Introduction

Observing facilities like Atacama Large Millimeter/sub–millimeter Array (ALMA) are used for observing a wide range of objects, from the Sun to the stars and galaxies. Those facilities usually operate in equatorial (R.A., Dec.) coordinate system which is not of much use to solar physicists who are accustomed working in heliographic/heliocentric coordinate systems (Thompson 2006). Presented in this note are methods for conversion of equatorial to helioprojective coordinates and vice versa. Although the problem might seem trivial (it basically transforms to the calculation of angular distance and position angle), there are several things that need to be dealt with care (see below and e.g. Meeus (1998)). Moreover, by the number of questions I received regarding this problem from other researchers, there seems to be a lack of documentation on this.

2 Methods

The problem can be put this way: given equatorial coordinates of the target (specified as right ascension, α, and declination, δ), the equatorial coordinates of the solar center (α_0, δ_0) and the position angle of the solar north pole (P angle,

*ivica.skokic@gmail.com

measured from celestial north towards east), find the helioprojective coordinates (θ_x, θ_y, sometimes also called solar x, y) of the target. Also for the reverse problem, if (θ_x, θ_y) are given, find the target (α, δ).

2.1 Approximate method

Since the Sun is only ~ 0.5 degrees in diameter when viewed from Earth, it is possible to consider the small angle approximation and planar geometry for targets near the Sun. In that case, the conversion between equatorial and helioprojective coordinates is just a rotation of the differences of equatorial coordinates of the Sun and the target by the solar P angle, corrected for the R.A. shrinkage with declination ($\cos\delta_0$ term):

$$\begin{aligned}\theta_x &= -(\alpha-\alpha_0)\cos\delta_0\cos P + (\delta-\delta_0)\sin P \\ \theta_y &= (\alpha-\alpha_0)\cos\delta_0\sin P + (\delta-\delta_0)\cos P\end{aligned} \tag{1}$$

Helioprojective coordinates (θ_x, θ_y) are usually measured in arcseconds (positive towards solar west and north, hence the minus sign in the equation for θ_x). Converting back to equatorial coordinates is straightforward:

$$\begin{aligned}\alpha &= \frac{-\theta_x\cos P + \theta_y\sin P}{\cos\delta_0} + \alpha_0 \\ \delta &= \theta_x\sin P + \theta_y\cos P + \delta_0\end{aligned} \tag{2}$$

This method should work for small angular distances from the solar center, say up to 10 arcminutes (Meeus 1998). Numerical tests show that error of ~ 1 arcsec is expected for targets on the solar limb. It starts to fail badly at angular distances larger than 1 degree and around poles (Sinnott 1984), so a better method is needed.

2.2 Exact method

By applying spherical geometry methods it is possible to derive exact equations for the transformation. Consider the Sun (S) and the target (T) on the celestial sphere as viewed from inside the sphere, with celestial north denoted as N (Figure 1a). Those points define a spherical triangle with sides $(90^\circ-\delta)$, $(90^\circ-\delta_0)$ and ρ. ϕ is the position angle of the target with respect to the solar center and ρ is an angular distance between solar center and the target. An application of the law of (co)sines on this spherical triangle gives:

$$\begin{aligned}\cos\rho &= \cos\delta\cos\delta_0\cos(\alpha-\alpha_0) + \sin\delta\sin\delta_0 \\ \tan\phi &= \frac{\sin(\alpha-\alpha_0)}{\tan\delta\cos\delta_0 - \sin\delta_0\cos(\alpha-\alpha_0)}\end{aligned} \tag{3}$$

When calculating the arctanget of the second equation, care must be taken of the correct quadrant, so it is best to use "atan2/arctan2" functions. Although

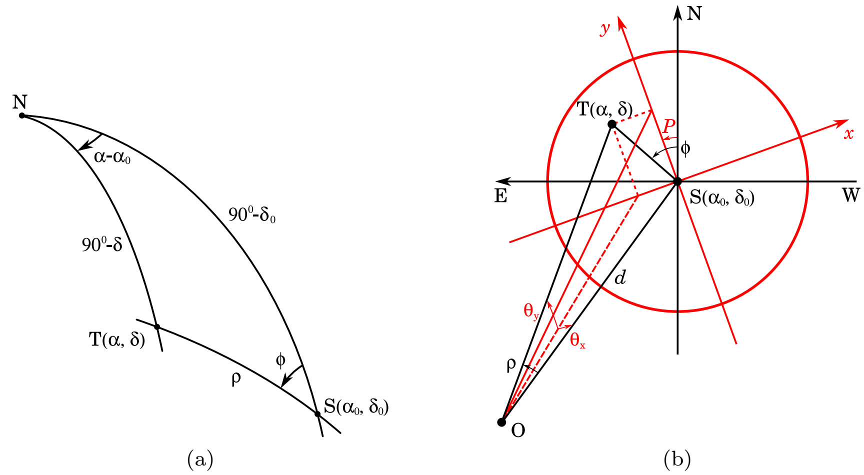

Figure 1: a) A spherical triangle on the celestial sphere defined by Sun center (S), target (T) and celestial north (N). b) Target (T) located on the solar disc (red circle) with helioprojective coordinate system (in red) and equatorial coordinate system (black). Observer (O) is located at a distance d from the Sun.

equations (3) are exact, in practice numerical problems can arise because computers have limited precision (rounding errors). The problem lies in the equation (3) for angular distance ρ because the cosine changes very slowly and has a value close to one for very small angular distances (Sinnott 1984; Meeus 1998). Numerical tests using double precision arithmetic on modern computers show that the error in using upper equations is less than 0.1 arcsec, which is satisfactory for most applications. However, for high precision calculations a better approach is needed. To address this problem, Sinnott (1984) proposed the use of the haversine function (hav $x = \sin^2(x/2)$) which takes care of most problems but still has rounding errors for special antipodal points. The equation has the form:

$$\sin\frac{\rho}{2} = \sqrt{\sin^2\frac{\delta-\delta_0}{2} + \cos\delta\cos\delta_0\sin^2\frac{\alpha-\alpha_0}{2}} \tag{4}$$

An even better approach is to use the equation derived for angular distances on the ellipsoid by Vincenty (1975), adapted to circle by setting equal axes:

$$\tan\rho = \frac{\sqrt{[\cos\delta\sin(\alpha-\alpha_0)]^2 + [\cos\delta_0\sin\delta - \sin\delta_0\cos\delta\cos(\alpha-\alpha_0)]^2}}{\sin\delta\sin\delta_0 + \cos\delta\cos\delta_0\cos(\alpha-\alpha_0)} \tag{5}$$

Although more complicated, this one should be numerically stable for all angles and distances.

Helioprojective coordinates are calculated from (Figure 1b, note that triangle $\triangle$ OST is a right triangle with the right angle $\measuredangle$ OST at point S):

$$\begin{aligned} \tan\theta_x &= -\tan\rho\sin(\phi - P) \\ \sin\theta_y &= \sin\rho\cos(\phi - P) \end{aligned} \tag{6}$$

Inverse equations are:

$$\begin{aligned} \tan(\phi - P) &= \frac{-\sin\theta_x}{\tan\theta_y} \\ \cos\rho &= \cos\theta_x\cos\theta_y \\ \sin\rho &= \sqrt{\sin^2\theta_x\cos^2\theta_y + \sin^2\theta_y} \end{aligned} \tag{7}$$

In upper equations, the numerical round-off error problem arises again in the cosine equation for ρ. To avoid it, one can use the sine equation or divide the two to get the tangent equation.

By applying spherical geometry equations on Figure 1a, we find the target equatorial coordinates:

$$\begin{aligned} \sin\delta &= \sin\delta_0\cos\rho + \cos\delta_0\sin\rho\cos\phi \\ \tan(\alpha - \alpha_0) &= \frac{\sin\rho\sin\phi}{\cos\rho\cos\delta_0 - \sin\rho\sin\delta_0\cos\phi} \end{aligned} \tag{8}$$

3 Conclusion

The approximate method should be precise enough for targets not too far from the disc center, but it starts to fail near the solar limb. Exact method with equations (3) has an opposite behavior, meaning that precision is getting worse for targets very close to the solar center, when used in practice due to rounding errors. However, with today's typical use of double precision in computer languages, errors smaller than 0.1 arcsec are expected. It is possible to use the combination of the two methods, approximate for targets near the center, and exact for more distant targets, but better approach for transformation of equatorial to helioprojective coordinates would be to use haversine or Vincenty equations to calculate ρ, as they do not have those numerical problems. The reverse problem of going from helioprojective to equatorial coordinates is not affected that much by rounding errors since sine or tanget can be used to get ρ.

Revision History

Revision	**Date**	**Author(s)**	**Description**
0.1	April 8, 2017	I.S.	created

0.2	October 30, 2019	I.S.	corrected mislabeled θ_y in Fig. 1b and corrected Eqs. (1), (6) and (7). A lot of text corrections too.

References

Meeus, J. 1998, Astronomical algorithms (Willmann-Bell)

Sinnott, R. W. 1984, Sky & Telescope, 68, 158

Thompson, W. T. 2006, A&A, 449, 791

Vincenty, T. 1975, Survey Review, 23, 88